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Modified Hagedorn formula including temperature fluctuation: Estimation of temperatures at RHIC experiments

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Abstract. We have systematically estimated the possible temperatures obtained from an analysis of recent data on p_t distributions observed at RHIC experiments. Using the fact that the observed p_t distributions cannot be described by the original Hagedorn formula in the whole range of transverse momenta (in particular above 6 GeV/c), we propose a modified Hagedorn formula including temperature fluctuation. We show that by using it we can fit p_t distributions in the whole range and can estimate consistently the relevant temperatures, including their fluctuations.

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1 Introduction

One of the characteristic features in every high energy collision experiment is the production of large numbers of secondaries (mostly pions). From the very beginning of the history of the multiparticle production processes, it was realized that a possible way to treat them was to employ some sort of statistical approach [1–4]. This idea found its most mature formulation in the statistical bootstrap model proposed by Hagedorn [5–8], in which the exponential growth of the number of hadronic resonances with mass is one of the most fundamental issues [9, 10]; for a recent review, see [11]. The proposed formula is

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}p^3} = C\int \mathrm{d}m\rho(m)\exp\left(-\sqrt{p_l^2+p_t^2+m^2}\beta_0\right). \quad (1)$$

In (1), $\rho(m)$ denotes the density of resonances given by

$$\rho(m) = \frac{\exp(m\beta_H)}{(m^2+m_0^2)^{5/4}}, \quad (2)$$

where $\beta_H = 1/(k_B T_H)$, the inverse of the so called Hagedorn temperature T_H, is a parameter to be deduced from data on resonance production [12, 13]. The other parameter is $\beta_0 = 1/(k_B T_0)$, with T_0 explicitly governing the observed energy distribution and therefore identified with the *temperature of the hadronizing system*. In the following we put $k_B = 1$. One of the aims in the study of multiparticle production processes is therefore the best possible estimation of this quantity. To this end we would like to investigate the measured transverse momentum (p_t) distributions integrated over the longitudinal degrees of freedom. From (1), we have

$$S_0 \equiv \frac{\mathrm{d}^2\sigma}{2\pi p_t\,\mathrm{d}p_t} = C\int \mathrm{d}m\rho(m)m_t K_1(m_t\beta_0), \quad (3)$$

where $m_t = \sqrt{p_t^2+m^2}$ is the transverse mass, and K_1 is the Bessel function.

However, as has recently been demonstrated by us [14], this simple formula can explain the RHIC data only in the limited range of transverse momenta, namely for $p_t \le 6$ GeV/c. For larger values of p_t the data exhibit a power-like tail. There are many attempts to explain it using some kind of nonequilibrium approach, like, for example, the flow or decay of resonances (see [15] for a recent review and further references); instead of trying to exclude them we would like to investigate the possibility that the observed nonexponential spectra could result from some form of equilibrium characteristic of nonextensive thermodynamics. In fact, as was shown in [14], using an approach based either on nonextensive statistics or on a stochastic approach one can successfully account for the whole range of

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the observed transverse momenta. The reason for this success is the fact that in both approaches the resultant distributions are *intrinsically non-exponential*, ranging from a power-law-like form (cf. (6) below) to a gaussian in transverse rapidity [16] (which can be regarded as another implementation of the effective power-law distribution)[1]. The fact that the proposed formulas can fit *the whole range* of p_t is by itself very interesting and an important observation, as it shows that the power law is present not only in very hard scale physics but that it reflects also a possible nontrivial property of hadronic matter in equilibrium (like, for example, existence of a quark–gluon plasma) [18, 19].

Such properties are best seen in an approach using a nonextensive statistical model in which two parameters are used: the action of the heat bath is described now by the mean temperature T_0 and by the nonextensivity parameter q, which can be identified with some specific intrinsic fluctuations of the temperature existing in the hadronizing system under consideration [20]. In the case when these fluctuations can be described by a gamma distribution one can write exact formulas [20] telling us that [20]

$$[1-(1-q)\beta_0 H_0]^{\frac{1}{(1-q)}} = \int_0^\infty \mathrm{e}^{-\beta H_0} f_\Gamma(\beta)\,\mathrm{d}\beta\,, \quad (4)$$

where

$$f_\Gamma(\beta) = \frac{1}{\Gamma(\alpha)}\left(\frac{\alpha}{\beta_0}\right)^\alpha \beta^{\alpha-1}\mathrm{e}^{-\frac{\alpha}{\beta_0}\beta}\,, \quad \alpha = \frac{1}{q-1}\,. \quad (5)$$

In general, one refers to the concept of the so called *superstatistics* introduced in [21]. In our previous work [14], RHIC data were described by the following distribution with $H_0 = \sqrt{p_l^2 + m_\mathrm{t}^2}$:

$$\frac{\mathrm{d}^2\sigma}{2\pi p_\mathrm{t}\,\mathrm{d}p_\mathrm{t}} = C\int_0^\infty \mathrm{d}p_l \left[1 - \frac{1-q}{T_0}\sqrt{p_l^2+m_\mathrm{t}^2}\right]^{\frac{1}{1-q}}\,. \quad (6)$$

As is seen in [14], this formula leads to very good agreement with all RHIC data [22–24].

It is important to notice that (6) has essentially the same form as the formula proposed a long time ago and used with success in many QCD-inspired power-law fits to experimental data [25–28] (recently used also by RHIC collaborations [29]):

$$\left(1+\frac{p_\mathrm{t}}{p_0}\right)^{-n} \longrightarrow \begin{cases} \exp\left(-\frac{n}{p_0}p_\mathrm{t}\right) & \text{for } p_\mathrm{t}\to 0,\\ \left(\frac{p_0}{p_\mathrm{t}}\right)^n & \text{for } p_\mathrm{t}\to\infty. \end{cases} \quad (7)$$

However, one has also to realize the important difference in the physical pictures leading to (6) and (7). The underlying physical picture in (7) is that the small p_t region is governed by *soft physics*, described by some unknown unperturbative theory or model, and the large p_t region is governed by *hard physics*, believed to be described by perturbative QCD. Contrary to it, the nonextensive formula (6), which is valid in the whole range of p_t, does not claim to originate from any particular theory. It merely offers the kind of general unifying principle, namely the existence of some kind of complicated equilibrium involving all scales of p_t, which is described by two parameters, T_0 and q: the temperature T_0 describing its mean properties, and the parameter q describing the action of the possible nontrivial long-range effects believed to be caused by fluctuations but essentially also by some correlations or long memory effects [30, 31][2].

2 Calculations and results

In this paper, we would like to compare the results of an analysis of p_t spectra measured at RHIC experiments [22–24] performed by using three approaches: the original Hagedorn model, (3), the QCD-inspired power-like formula, (7), and the modified Hagedorn formula including a temperature fluctuation given by

$$S_\mathrm{tot} \equiv \frac{\mathrm{d}^2\sigma}{2\pi p_\mathrm{t}\,\mathrm{d}p_\mathrm{t}} = C\int \mathrm{d}y\cosh y \int \mathrm{d}m\rho(m)m_\mathrm{t} \times [1-\beta_0(1-q)m_\mathrm{t}\cosh y]^{\frac{1}{1-q}}\,. \quad (8)$$

It can be written also in the form of a series ($\alpha = 1/(q-1)$):

$$S_\mathrm{tot} = \frac{4C}{\alpha-1}\int_{m_\pi}^\infty \mathrm{d}m\rho(m)\frac{\beta_0 m_\mathrm{t}^2/\alpha}{(1+\beta_0 m_\mathrm{t}/\alpha)^\alpha} \times\sum_{k=0}^\infty \frac{\Gamma(k+3/2)\Gamma(\alpha+1+k)}{\Gamma(\alpha+k+1/2)\Gamma(k+1)}\left(\frac{1-\beta_0 m_\mathrm{t}/\alpha}{1+\beta_0 m_\mathrm{t}/\alpha}\right)^k ; \quad (9)$$

or, accounting for the smallness of $q-1$ encountered in our fits and of the fact that we are interested only in the midrapidity region (i.e., for small y) one can write it also as[3]

$$S_\mathrm{tot} \simeq C\int \mathrm{d}y\cosh y\int \mathrm{d}m\rho(m)m_\mathrm{t} \times\left[1+\frac{1}{2}(q-1)\beta_0^2 m_\mathrm{t}^2 y^2\right] \times\exp\left[-\beta_0 m_\mathrm{t}\cosh y + \frac{1}{2}(q-1)\beta_0^2 m_\mathrm{t}^2\right]\,. \quad (10)$$

Equations (9) and (10) are used to check the numerical integration of (8).

[1] See also [17], where the flow effect is included and the relation between a Gaussian-like distribution in transverse rapidity and a power-law behavior in p_t is discussed.

[2] The origin of such fluctuations and/or correlations must most probably be traced back to the nonperturbative QCD, cf., for example, [32, 33].

[3] In our case, because we are integrating over the whole mass spectrum $\rho(m)$ in the Hagedorn formula, we cannot simply expand in $(q-1)$ and keep only the linear term as it is done on such occasions in the literature, cf., for example [34], because for large masses m the series becomes divergent.

Table 1. Parameters of our analysis presented in Fig. 1 (left panel) by the use of (3), which corresponds to $q-1=0$ in (8). Those for the right panel with $q-1 \neq 1$ in (8) can be found in Table 4. Other parameters are $m_0 = 0.5$ GeV (fixed), $\delta T_{\rm H} = 0.0001$–$0.002$ and $\delta T_0 = 0.0001$–0.002. Notice that very large values of χ^2 are obtained for fits with $q-1=0$

C.C. [%]	C	$T_{\rm H}$ [GeV]	T_0 [GeV]	χ^2/n.d.f.
0–5	816 ± 15	0.086	0.085	532/32
20–30	382 ± 7	0.077	0.076	249/32
60–80	106 ± 2	0.037	0.037	308/32

At first the STAR data [22] were analyzed using (3) (which corresponds to $q = 1$ in (8)) and the modified Hagedorn formula, (8). Results are shown in Fig. 1 and Table 1. The corresponding results for the BRAHMS and PHENIX data [23, 24] are very similar. As can be seen in Fig. 1, whereas distributions in the small $p_{\rm t}$ region can be explained by the simple formula (3), data including the larger $p_{\rm t}$ region can only be explained by using the modified Hagedorn formula, (8) (or (9) and (10)). The nonzero values of $|q-1|$ are then interpreted as an indication of the sizeable temperature fluctuations existing in the hadronizing system [20, 21].

In fact RHIC data allow us to investigate the temperature fluctuation in more detail; cf. Fig. 2. The centrality cut region, C.C. = 0–5%, of the STAR data [22] was fitted by using, respectively, (3) (with $q-1=0$, left panel) and (8) (with q as in Table 4, right panel). A fit was performed by fixing all parameters in (3) and (8) except β_0; β $(=\beta_0)$ is then calculated for each of 35 data points, and it is assumed that the reciprocal of each error bar calculated by the fitting program MINUIT is proportional to the corresponding probability of this value of β, $P(\beta)$. In this way a probability distribution for β is obtained and presented in the form of a histogram in Fig. 2. The histogram in each panel is then fitted to the Gamma distribution with $\alpha = 55\,000$ (shown as solid curves). The mean value $\langle\beta\rangle$ is also shown in each panel in Fig. 2. As can be seen, a good fit can be obtained *only* when (8) is used, and in this case the resultant distribution of temperatures is very narrow. This result suggests that accounting for intrinsic fluctuations considerably narrows the distribution of the temperatures (actually its reverse, $\beta = 1/T$) and minimizes what can be regarded as a kind of systematic error in the deduction of β_0 from the experimental data. Therefore it strongly suggests that the modified Hagedorn formula, (8), should be used whenever possible.

The results of our fits to the RHIC data [22–24], performed by using (6) (as given by nonextensive statistical approach), (7) (representing the QCD-inspired power-law formula) and (8) (given by the modified Hagedorn formula proposed by us here) are presented in, respectively, Tables 2–4. The results for the STAR data are also shown in Fig. 3. In particular, the left hand panels of Fig. 3 demonstrate the contribution of the different mechanisms represented, respectively, by $S_{\rm tot}$ and S_0. It is clear that the data for the larger $p_{\rm t}$ region can be explained only by $S_{\rm tot}$,

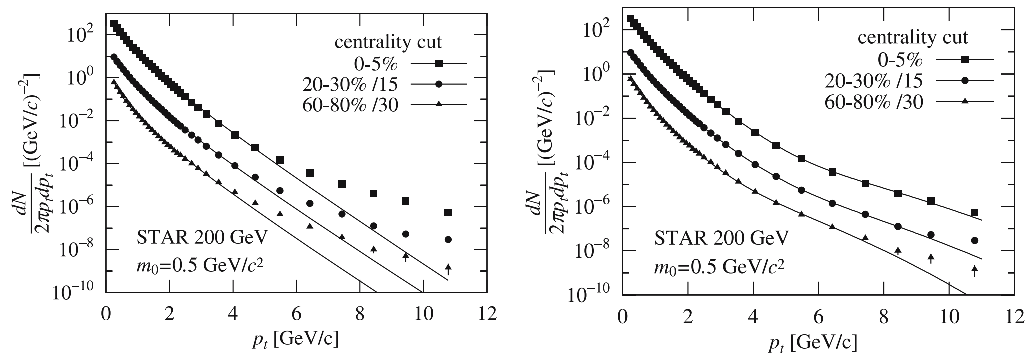

Fig. 1. Analysis of the STAR data [22] by using the usual Hagedorn formula ((3), *left panel*) and its nonextensive generalization ((8), *right panel*)

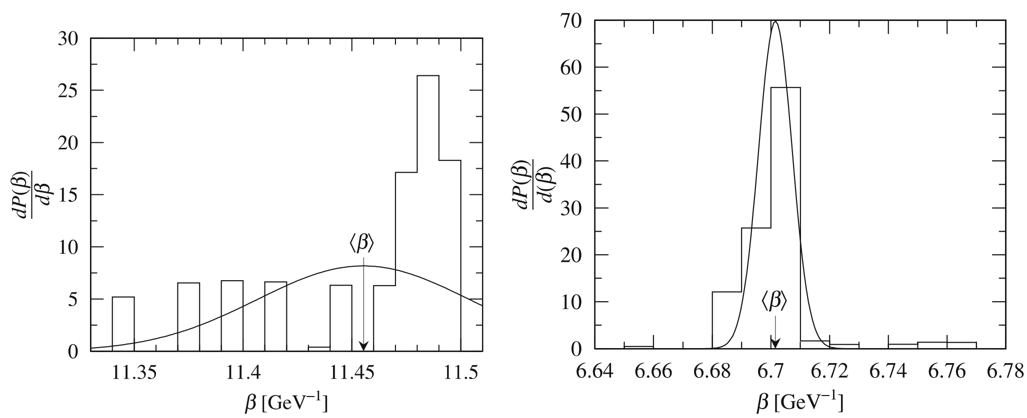

Fig. 2. Temperature fluctuations in the STAR data (for C.C. = 0–5%) [22] are analyzed by the use of (3) with $q-1=0$ (*left panel*) and (8) with $q-1 \neq 0$ (*right panel*)

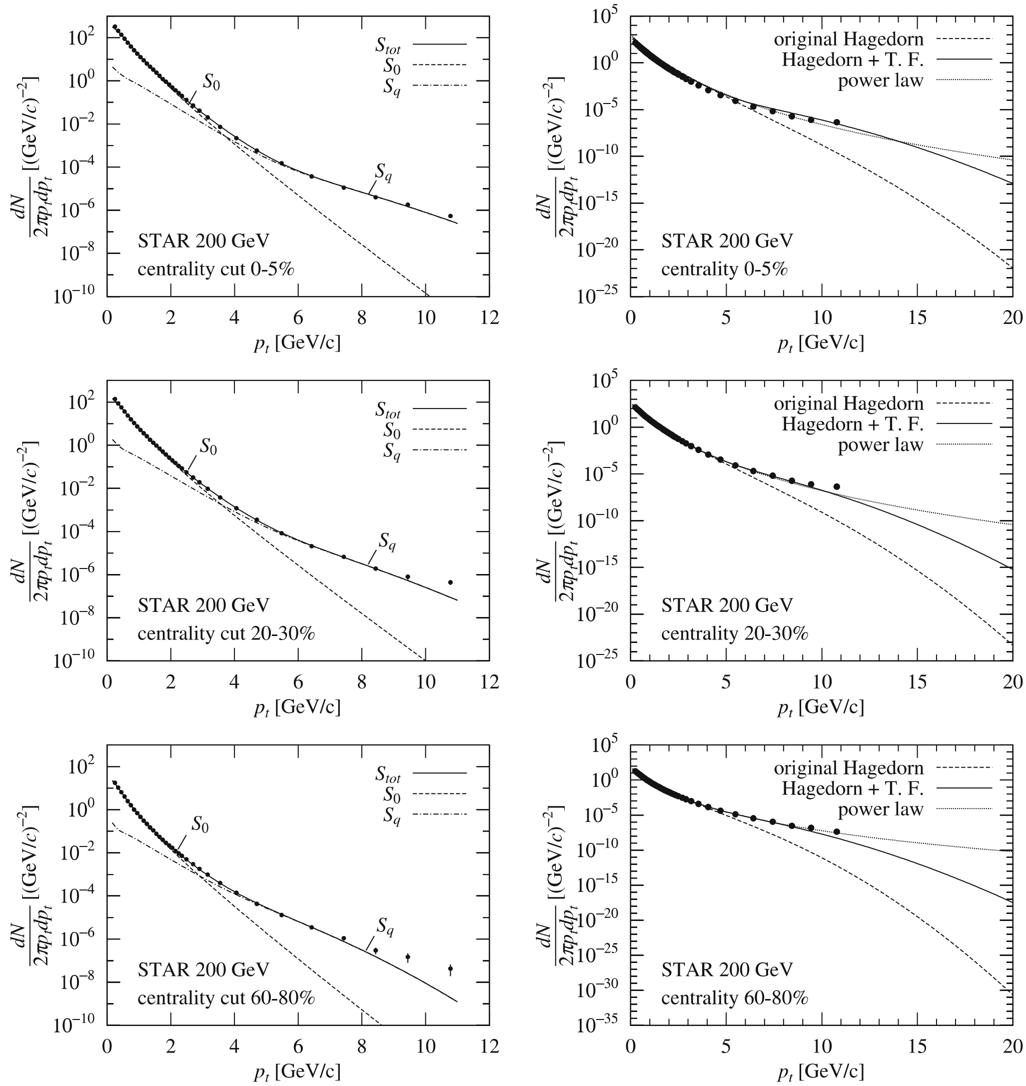

Fig. 3. Example of the visualization of results presented in Tables 2 and 4 using STAR [22] results. The *left panels* show in detail the contribution of the different mechanisms represented by $S_{\rm tot}$, (8), by S_0 (3) and by their difference denoted by $S_q = S_{\rm tot} - S_0$. The *right panels* show the results on the original Hagedorn model, (1), by a *dashed line*, the modified Hagedorn formula including temperature fluctuations, (8), by a *solid line*, and (7) by a *dotted line*

Table 2. Analysis of the RHIC data [22–24] by means of the nonextensive approach as given by (6)

Coll.	C.C.	c	T_0 [GeV]	q	$1/(q-1)$	χ^2/n.d.f.
BRAHMS	0%–10%	936 ± 68	0.227 ± 0.005	1.0394 ± 0.0026	25.4	10.2/23
	10%–20%	716 ± 56	0.217 ± 0.005	1.0455 ± 0.0029	22.0	12.9/23
	20%–40%	468 ± 41	0.208 ± 0.006	1.0507 ± 0.0033	19.7	12.8/23
	40%–60%	265 ± 32	0.185 ± 0.007	1.0607 ± 0.0044	16.5	10.6/23
	60%–80%	36.2 ± 4.2	0.165 ± 0.005	1.0764 ± 0.0024	13.1	2.76/23
PHENIX	0%–5%	1530 ± 359	0.195 ± 0.012	1.0461 ± 0.0060	21.7	5.00/29
	5%–15%	1200 ± 276	0.193 ± 0.012	1.0472 ± 0.0057	21.2	3.56/29
	15%–30%	760 ± 180	0.189 ± 0.012	1.0503 ± 0.0058	19.9	5.50/29
	30%–60%	384 ± 96	0.170 ± 0.011	1.0613 ± 0.0055	16.3	2.60/29
	60%–80%	120 ± 39	0.144 ± 0.012	1.0728 ± 0.0067	13.7	10.5/29
	80%–92%	59.2 ± 32.0	0.114 ± 0.017	1.0879 ± 0.0106	11.4	8.99/29
STAR	0%–5%	3980 ± 186	0.164 ± 0.002	1.0651 ± 0.0009	15.4	172/32
	5%–10%	2900 ± 148	0.169 ± 0.002	1.0622 ± 0.0011	16.1	64.5/32
	10%–20%	2340 ± 114	0.164 ± 0.002	1.0662 ± 0.0011	15.1	66.4/32
	20%–30%	1630 ± 81	0.162 ± 0.002	1.0684 ± 0.0011	14.6	40.7/32
	30%–40%	1170 ± 61	0.158 ± 0.002	1.0709 ± 0.0011	14.1	38.9/32
	40%–60%	739 ± 39	0.146 ± 0.002	1.0772 ± 0.0010	13.0	14.7/32
	60%–80%	328 ± 19	0.130 ± 0.002	1.0850 ± 0.0011	11.8	9.39/32
	pp (nsd)	49.9 ± 5.5	0.111 ± 0.003	1.0894 ± 0.0014	11.2	10.1/29

Table 3. Analysis of the RHIC data [22–24] by means of the QC-inspired power-like formula (7). For comparison the results for pp collisions are also shown

Coll.	C.C.	c	n	p_0 [GeV]	$\tilde{T}=p_0/n$	χ^2/n.d.f.
BRAHMS	0%–10%	353 ± 19	32.2 ± 3.3	8.89 ± 1.09	0.276	7.96/23
	10%–20%	260 ± 15	26.4 ± 2.5	7.05 ± 0.83	0.267	14.7/23
	20%–40%	163 ± 11	22.8 ± 2.1	5.87 ± 0.70	0.257	13.9/23
	40%–60%	83.7 ± 7.5	17.9 ± 1.8	4.13 ± 0.56	0.231	11.7/23
	60%–80%	11.1 ± 1.0	12.8 ± 0.5	2.58 ± 0.17	0.202	2.86/23
PHENIX	0%–5%	536 ± 398	23.8 ± 34.0	5.54 ± 31.26	0.233	4.69/29
	5%–15%	417 ± 276	23.0 ± 23.7	5.30 ± 21.15	0.231	3.58/29
	15%–30%	260 ± 149	21.3 ± 13.2	4.84 ± 11.01	0.227	5.54/29
	30%–60%	120 ± 924	16.6 ± 91.8	3.40 ± 90.58	0.205	2.66/29
	60%–80%	32.1 ± 38.2	13.5 ± 31.5	2.38 ± 30.46	0.177	10.3/29
	80%–92%	12.8 ± 35.4	10.8 ± 31.6	1.53 ± 30.44	0.142	8.83/29
STAR	0%–5%	1140 ± 41	15.4 ± 0.3	3.10 ± 0.09	0.201	194/32
	5%–10%	843 ± 33	16.5 ± 0.4	3.44 ± 0.12	0.208	68.6/32
	10%–20%	660 ± 25	15.3 ± 0.3	3.12 ± 0.10	0.203	72.3/32
	20%–30%	457 ± 18	14.7 ± 0.3	2.94 ± 0.09	0.200	42.8/32
	30%–40%	319 ± 13	14.1 ± 0.3	2.77 ± 0.09	0.196	38.1/32
	40%–60%	190 ± 8	12.6 ± 0.2	2.30 ± 0.07	0.182	13.9/32
	60%–80%	75.4 ± 3.3	11.3 ± 0.2	1.84 ± 0.06	0.163	7.18/32
	pp (nsd)	10.8 ± 0.9	10.4 ± 0.2	1.42 ± 0.06	0.136	11.6/29

Table 4. Analysis of RHIC data [22–24] by means of nonextensive modification of the Hagedorn formula as given by (8). Maximum m is fixed at 70 GeV (therefore in (9) one always has $(1-\beta_0 m_t/\alpha)>0$). Numbers of divisions for y and m in the computations are given in the last column

Coll.	C.C.	c	$q-1$	T_H [GeV]	T_0 [GeV]	χ^2/n.d.f.	no. of div.
BRAHMS	0%–10%	156 ± 3	0.00	0.192 ± 0.000	0.178 ± 0.000	15.4/22	6×6
	10%–20%	106 ± 5	$(4.76\pm0.62)\times10^{-4}$	0.206 ± 0.007	0.187 ± 0.005	13.1/22	6×7
	20%–40%	67.7 ± 4.9	$(8.49\pm21.05)\times10^{-5}$	0.177 ± 0.013	0.166 ± 0.010	11.6/22	5×3
	40%–60%	32.5 ± 2.9	$(2.57\pm0.63)\times10^{-4}$	0.168 ± 0.010	0.157 ± 0.008	9.54/22	5×4
	60%–80%	5.00 ± 0.14	$(8.12\pm0.45)\times10^{-5}$	0.124 ± 0.000	0.120 ± 0.000	3.19/22	6×6
PHENIX	0%–5%	226 ± 56	$(1.21\pm2.31)\times10^{-4}$	0.16 ± 0.02	0.152 ± 0.019	4.98/29	5×6
	5%–15%	157 ± 34	$(4.01\pm0.02)\times10^{-4}$	0.183 ± 0.023	0.167 ± 0.017	3.32/29	6×5
	15%–30%	87.5 ± 10.4	$(4.26\pm0.80)\times10^{-4}$	0.187 ± 0.010	0.170 ± 0.008	4.31/29	5×3
	30%–60%	50.3 ± 8.7	$(1.64\pm0.47)\times10^{-4}$	0.140 ± 0.012	0.133 ± 0.010	2.54/29	6×7
	60%–80%	27.8 ± 2.5	$(1.99\pm0.43)\times10^{-5}$	0.0731 ± 0.0002	0.0719 ± 0.0002	9.91/29	5×6
	80%–92%	10.0 ± 1.2	$(1.24\pm0.30)\times10^{-5}$	0.0565 ± 0.0002	0.0558 ± 0.0001	8.71/29	12×12
STAR	0%–5%	477 ± 13	$(1.48\pm0.05)\times10^{-4}$	0.140 ± 0.001	0.132 ± 0.001	56.6/31	6×6
	5%–10%	443 ± 15	$(1.08\pm0.06)\times10^{-4}$	0.127 ± 0.002	0.122 ± 0.002	38.0/31	7×6
	10%–20%	326 ± 17	$(1.02\pm0.10)\times10^{-4}$	0.126 ± 0.004	0.121 ± 0.003	33.8/31	6×5
	20%–30%	236 ± 14	$(8.15\pm1.00)\times10^{-5}$	0.119 ± 0.004	0.115 ± 0.004	30.0/31	6×6
	30%–40%	169 ± 10	$(7.13\pm0.09)\times10^{-4}$	0.113 ± 0.004	0.109 ± 0.004	25.5/31	6×6
	40%–60%	109 ± 4	$(4.40\pm0.22)\times10^{-5}$	0.0961 ± 0.0014	0.0937 ± 0.0013	24.5/31	6×7
	60%–80%	46.0 ± 1.0	$(2.80\pm0.08)\times10^{-5}$	0.0797 ± 0.0001	0.0782 ± 0.0000	23.4/31	6×7
	pp (nsd)	4.98 ± 0.15	$(2.87\pm0.08)\times10^{-5}$	0.0725 ± 0.0000	0.0711 ± 0.0000	38.2/28	20×22

which can be attributed to the intrinsic primordial temperature fluctuations in the hadronizing system. However, at present it is difficult to treat this as a possible signal of a quark–gluon plasma. Notice that the temperature parameter $T_0=1/\beta_0$ in Tables 2 and 4 was estimated by the use of (8) from the whole region of the transverse momenta, whereas $\tilde{T}_0=p_0/n$, which corresponds to the temperature in (7), shown in Table 3, governs only the small p_t region. The RHIC data show that we always have $\tilde{T}_0>T_0$, i.e., that inclusion of the fluctuations and long-range correlations present in the hadronizing system lowers the estimated value of its mean temperature. From Table 4, we can see that both temperatures, T_H and T_0, estimated by the use of (8) decrease as the centrality cut, C.C., increases (i.e., it can be argued that they increase with the volume of interaction; a similar effect concerning T_{H} has also been found in [35]). It should be emphasized that when one uses the modified Hagedorn formula, (8), then $T_{\mathrm{H}} \sim T_0 \sim m_\pi$, i.e., the estimated values of T_{H} and T_0 are almost equal to m_π, which we regard as a very reasonable result[4].

3 Summary

We have presented a systematic analysis of the RHIC data [22–24] on transverse momenta distributions, which allow, in principle, for the deduction of the parameter believed to represent the temperature T_0 of the hadronizing system. We have shown that in order to fit the whole range of p_{t}, one has to use a nonextensive approach, which accounts for the temperature fluctuations present in the hadronizing system. This has been compared with the approach using the old QCD-inspired power-like formulas introduced a long time ago. We have demonstrated that gradual accounting for the intrinsic dynamical fluctuations in the hadronizing system by switching from (6) (as given by a nonextensive statistical approach) to the modified Hagedorn formula including temperature fluctuations, (8), substantially lowers the values of the parameter $q-1$. This is because part of the fluctuations ascribed in (6) to q are accounted for by the resonance spectrum $\rho(m)$ present in the Hagedorn formula. It also changes the temperature we are looking for. Therefore one has to be very careful when interpreting the temperature parameter obtained in such fits, especially when attempting to address any questions concerning quark–gluon plasma production issues[5]. If data with larger p_{t} are available, we can further investigate whether the modified Hagedorn formula including temperature fluctuations is really applicable or not.

One more remark is in order here. The T_0 parameters obtained by us are in a range favorable for $T_{\mathrm{c}} = 170\,\mathrm{MeV}$, the QCD crossover temperature. Traditional exponential fits for the low p_{T} part of the pion spectra, on the other hand, used to give $T = 340\,\mathrm{MeV}$, pointing out a transverse flow with a Doppler blue-shift factor of two. However, we do not claim that there is no transverse flow in the RHIC experiments; we only show that a nonextensive approach can mimick this effect as well.

[4] Actually, the analysis performed assuming both thermal and chemical equilibrium and including also baryons performed by the GSI group [36] gives $T = 170\,\mathrm{MeV}$. In our case we are considering only pions and get $T \simeq m_\pi$. This difference is important for the description of the phase diagram, and we plan to address it elsewhere. One more remark is in order here. The T_0 parameters obtained by us are in the range of $T_{\mathrm{c}} = 170\,\mathrm{MeV}$, the QCD crossover temperature. On the other hand, traditional exponential fits for the low p_{T} part of the pion spectra used to give $T = 340\,\mathrm{MeV}$, pointing to a transverse flow with a Doppler blue-shift factor of two. However, we do not claim that there is no transverse flow in the RHIC experiments; we only show that a nonextensive approach can mimic this effect as well.

[5] One should be aware of the fact that there is still an ongoing discussion on the meaning of the temperature in nonextensive systems. However, the small values of the parameter q deduced from the data allow us to argue that, to first approximation, T_0 can be regarded as the hadronizing temperature in a such system. One must only remember that in general what we study here is not so much the state of equilibrium but rather some kind of stationary state. For a thorough discussion of the temperature of nonextensive systems, see [37]. It is also worth to be aware that in addition to the possibility of long-range correlations and memory effects to be at work in relativistic heavy-ion reactions (which have so far not yet been proven) one can also view $q > 1$ as a general leading order finite-size effect, $q = 1 + O(1/N)$, as proposed in [38, 39].

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